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Frequency Domain Analysis of Nigeria All Share and Capital Index from 1989-2010 (A Case Study of Nigerian Stock Market)

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Abstract: In recent years the method of wavelet analysis has been opened to researchers. It is the analyses of data at different level of decomposition and can capture the characteristics of data series in all decomposition level. In this research work, data was collected on the Nigerian stock index for All Share and Capital market indexes (1989-2010). The data were analyzed by wavelet method to detect the aberrant observations (AOs) over the period under study for the two indexes. Akaike information criterion (AIC) was also used to detect the 'best model' for the two indexes using some distributions. A total of seventeen and eleven AOs were detected from the original data collected on All Share and Capital Market indexes respectively. In the first, second and third resolutions, a total of four, two and two AOs were detected from the All Share index, while a total of five, four and three AOs were detected from that of Capital Market index. The results obtained showed the AOs detected in the analysis of the original data maintain the same or closely the same positions as that obtained from the analysis of the decomposed data for the two stock indexes. It was observed that the index of stocks in March, July and December are more and less in February, March, and November for the two indexes. The AIC results show that, the Cauchy distribution has the smallest AIC values among the distributions used, which means is the 'best model'.

Keywords: Aberrant, Resolution, Stock, Indexes, Decomposition

1. Introduction

1.1. Nigerian Stock Exchange (NSE)

The Lagos stock exchange (LSE) established in 1961, became the Nigeria stock Exchange (NSE) in 1977 as the hub of the capital market activities where medium and long-term financial securities are traded. NSE thus provides avenues whereby sellers and buyers exchange securities at mutually satisfactory prices, thereby creating liquidity through its price mechanism.

Initially NSE had a set of requirements to be fulfilled before a company is enlisted in the stock exchange market, but in 1985 another set of requirements for enlistment were issued to allow smaller and particularly wholly indigenous enterprises to be registered with the stock exchange: securities that met the initial requirements are referred to as First-Tier securities, whereas securities that could meet only the set of requirements are referred to as Second-Tier securities.

1.2. Aberrant Observations (AOS)

AOs are commonly encountered and their presence can seriously distort model-identification, parameter estimation and forecasting. Grubbs [7] defined an AO as "as an outlying observation", or AO is one that appears to deviate markedly from other members of a sample from which it occurs. In almost every (if not all) real data, there are presences of AOs; and most noticeable in large data sets, they are inevitable and are described as observation which are unusual, but not necessarily errors. Detection of AOs helps reveal important and valuable information from large data sets. In the field of meteorology, for example, spatial AOs can be associated with disastrous natural events such as tornadoes, hurricanes, and forest fires.

One of such fields where detection of AOs can be applied is the medical field. The number of patient coming to the hospital for medical attention can be viewed to follow a fluctuating pattern over time. These fluctuations can be as a result of AOs. AOs as pointed out earlier, does not necessarily connotes errors but contain valuable information about the data. Because of this, detecting the aberrant observation becomes paramount.

"An AO is an observation or a point that is considerably dissimilar or inconsistent with the remainder of the data" Ramasmawy et al. [12]. However, a data miner should be careful when automatically detecting and eliminating AOs because, if the data are correct, their elimination can cause the loss important hidden information Kantardzic, [10].

1.3. Causes of Aberrant Observations

In practical applications, data are easily subjected to malfunctions of the data collection mechanism, calculation errors or unaccepted extreme events. One of the resulting consequences is the appearance of AOs, which are defined in Barnett and Lewis [3] as observations with abnormal deviation from the mean of the remainder of the dataset. AOs can be due to several causes. The measurement can be incorrectly observed, recorded or entered into the process computer, the observed datum can come from a different population with respect to the normal situation and thus is correctly measured but represents a rare event. Hodge [9] pointed out that AOs are caused basically because of poor data quality (contamination), low quality measurements, malfunctioning equipment, manual error, and correct exceptional data.

1.4. Aim and Objectives of the Study

The aim of this study is to detect AOs in the All share index (ASI) and capital market index in Nigeria stock exchange with the following objectives to;

1) Use wavelet analysis to detect AOs in Nigeria stock index data set.
2) Compare the result obtained in (i) in terms of their positions at each resolution.
3) Use Akaike information criterion (AIC) to select the best model for the data set.

2. Wavelet Analysis

Wavelet analysis (also called wavelet theory or just wavelet) has attracted much attention recently in data processing. It has been successfully applied in many areas such as transient data analysis, image analysis, communication systems and other data processing applications. Most of the data in practice are time domain in their raw format. That is, whatever the data is measuring is a function of time Shittu and Aideyan [15]. Wavelet analysis or techniques provide multi-scale analysis of the data as a sum of orthogonal data corresponding to different time scales hence called time-scale analysis. It provides multilevel analysis to analyze the data at different levels. Bilen et al, [4].

The traditional way of analyzing a data in the frequency domain is well-known Fourier analysis which applies sinusoidal waves as the transformation filter Tian-Xiao [16]. The main drawback of this transformation is that it cannot maintain information of the time domain and will be unsuitable for data with irregular behavior such as spikes or data breaks. The wavelet transformation adopts a basis of spatially localized functions as its transformation filter Hodge [9]. Then based on wavelet of the original data through shifting and dilations, the wavelet transformation can capture the characteristics of data series both in the frequency domain and the time domain. It is an excellent tool for the analysis of the non-stationary data showing time-localized discontinuities or abrupt changes. By wavelet multi-resolution analysis (MRA) which combine resolutions from both time and frequency domains, data can be decomposed into different scales where the non-stationary of the data can be analysed according to their resolution levels; long run trends corresponds to the low frequency resolution and the spikes such as the outliers can be captured in the high frequency resolution. Shittu and Aideyan [15].

2.1. Definition of Wavelet. Eclay [6]

Let m $\in$ N. then for x $\in R, a$ function $\psi(\mathrm{x})$ is called a mother wavelet of order m if the following properties hold

$$w_\iota\text{: If m=0, } \psi(\mathrm{x}) \in L^\infty(\mathrm{R}).$$

If m$\geqslant$1, then $\psi(\mathrm{x})$ and all its derivation up to order m belong to L^∞(R).

w_ι:Expresses the regularity (i: e smoothness) of the wavelet.

$w_2 : \psi(x)$ and all its derivatives up to order m decrease rapidly as

$$x \to \pm\infty$$

w_3 : For each k ε (0,……, m),

w_2 and w_3 addresses the localization and oscillation of ψ

$$\int_{-\infty}^{\infty} x^k \, \psi(\mathrm{x}) dx = 0$$

w_4 the collection $(\psi_{j,k)})_{j,k\epsilon z}$ forms an orthonormal basis of L^2 (R), then being constructed from the mother wavelet using the identity.

w_4 is the dilation and transaction parameters respectively

$$\psi_{\mathrm{j,k}}\,(\mathrm{x}) = 2^{j/2}\psi(2^{\mathrm{j}}\mathrm{x} - \mathrm{k}), \mathrm{j}, \mathrm{k} \in \mathrm{z}$$

Called Haars wavelet proposed by Haar [8].

Fork=0

$$\psi_{j,}0^{(x)} = \psi[2^j(x)] j, k \in z$$

The bigger j is, the smaller the co-zero set $(0, {}^{1}\!/_{2^j})$ is where as the Fourier consist of only one choice of basis function of basis function $(1.\cos nx, \sin nx)$ Saravanan [14].

The earliest type of wavelet is the Haar wavelet. The Haar mother wavelet is a mathematical function defined by Sameh et al. [13]

$$\psi(x) = \begin{cases} 1 \; if \; 0 \leq x \leq 1/2 \\ -1 \; if \; 1/2 \leq x \leq 1 \\ 0 \; otherwise \end{cases}$$

2.2. The Haar Scaling Function

Two functions play a primary role in wavelet analysis, the scaling function ø and the wavelet function ψ. They both form a family of function that can be used to break or reconstruct or smothering a data set Aideyan et al, [1].

ø Is sometime called the father wavelet while ψ is the mother wavelet the simplest of wavelet analysis is based on the Haar scaling function. Percival [11]. Disadvantage of the Haar wavelet is that they are discontinuous therefore do not approximate continuous data set very well.

Definition: The Haar scaling function is defined as

$$\emptyset(x) = \begin{cases} 1 \; if \; 0 \leq x \leq 1 \\ 0 \; otherwise \end{cases}$$

For the purpose of this research work, Haar wavelet method was applied.

2.3. The Key Characteristics of Wavelet Analysis.

1. For a wide range of data as resolution decreases' including those with discontinuities guaranties the presence of required statistics. Sparsity of representation Aideyan and Shittu [1].
2. At a number of resolutions, it has the ability to analyze data and also to work with information at such resolutions.
3. Create localized features on synthesis has the ability to detect aberrant observations and represent neighbourhood features.
4. Efficiency in terms of compilation speed and storage.

2.4. Thresholding and Method of Analysis

Donoho et al. [5] propose a threshold τ base on the following result:

Let Z_i be independently, identical distributed (iid) standard normal random variables. We define

$$A_n = \{max_{i=1,n}|Z_i| \leq \sqrt{2logn}\}.$$

Then

$$\pi_n = \mathrm{P}(A_n) \rightarrow 0, \; \mathrm{n} \rightarrow \infty$$

In addition, if $B_n(\mathrm{t}) = \{max_{i=1,n}|Z_i| > t + \sqrt{2logn}\}$.

Then $\mathrm{P}(B_n(\mathrm{t})) < e^{\frac{t^2}{2}}$. That motivates the following threshold:

$$\tau^u = \hat{\sigma}\sqrt{2logn},$$

$$Z = \frac{x_j - \bar{x}_j}{\sigma_j \sqrt{2 \log n_j}}$$

Where Z is the threshold as proposed by Aideyan [2] and j= 1,2, ..., represents the level of wavelet decomposition.

Any value(s) obtained in Z is greater than 1 is/are regarded as aberrant observations.

To determine the exact position of the AO in the original data, we use the follows;

$$\bar{x}^* = \frac{1}{n-2} \sum_{t=2^i s_k, 2^i s_k - 1} X_t.$$

Where i is the number of resolution

The location of the outlier is $(2^i s_k - 1)$ if $|X_{2^i s_k} - \bar{X}^*| > |X_{2^i s_k - 1} - \bar{X}^*|$, Otherwise the location of

the AO in All Share index and Capital Market index are:

Table 1. Analysis of All Share index at different resolutions.

RESOLUTION	AO VALUE	AO POSITION	ACTUAL VALUE	DATE OF OCCURRENCE
256	1.01661896	212	47124	Apr-07
	1.103001397	213	49930.2	May-07
	1.146106423	214	51330.5	Jun-07
	1.19816614	215	53021.7	Jul-07
	1.114110876	216	50291.1	Aug. 2007
	1.11219927	217	50229	Sept. 2007
	1.11136198	218	50201.8	Oct. 2007
	1.2341265	219	54189.9	Nov. 2007
	1.351110028	220	57990.2	Dec. 2007
	1.234127116	221	54189.92	Jan. 2008
	1.586972681	222	65652.38	Feb. 2008
	1.505835002	223	63016.56	Mar. 2008
	1.395766811	224	59440.91	Apr-08
	1.38000945	225	58929.02	May-08
	1.288276363	226	55949	Jun-08
	1.200912266	227	53110.91	Jul-08
	1.037095176	228	47789.2	Aug. 2008
128	−1.243232705	102	33096.4	Aug. 2006
	−2.786480955	111	65652.38	Feb. 2008
	1.360143902	114	53110.91	Jul-08

RESOLUTION	AO VALUE	AO POSITION	ACTUAL VALUE	DATE OF OCCURRENCE
64	2.488832688	115	46216.13	Sept. 2008
	−2.728416621	56	63016.56	Mar. 2008
32	1.563345404	57	53110.91	Jul-08
	2.04816776	28	63016.56	Mar. 2008

Table 2. *Analysis of Capital Market index at different resolutions.*

RESOLUTION	AO VALUE	AO POSITION	ACTUAL VALUE	DATE OF OCCURRENCE
	1.111793399	219	8990.8	Nov. 2007
	1.298541383	220	10180.3	Dec. 2007
	1.378986069	221	10692.7	Jan. 2008
	1.663229818	222	12503.2	Feb. 2008
	1.603994833	223	12125.9	Mar. 2008
256	1.504364509	224	11491.3	Apr-08
	1.523706545	225	11614.5	May-08
	1.414719196	226	10920.3	Jun-08
	1.370807123	227	10640.6	Jul-08
	1.230122071	228	9744.5	Aug. 2008
	1.244628598	229	9836.9	Sept. 2008
	−1.3808688669	110	10180.3	Dec. 2007
	−2.1211973296	111	12503.2	Feb. 2008
128	1.105490295	114	106640.6	Jul-08
	2.263907637	115	9836.9	Sept. 2008
	−2.7455290782	127	7982.5	Oct. 2008
	1.072360453	55	10180.3	Dec. 2007
64	−2.144014665	56	12125.9	Mar. 2008
	1.39036062	58	7305.9	Nov. 2008
	−2.040243374	64	7913.8	Dec. 2010
	1.91636441	28	12125.9	Mar. 2008
32	−1.02700209	29	7305.9	Nov. 2008
	1.11080624	32	7913.8	Dec. 2010

Table 3. *AIC for all share index.*

Distribution	-2 LogL	Log Likelihood	AIC	RESOLUTION
Normal	5642.005	−2821.0025	5646.005	
Laplace	5622.592	−2811.296	526.592	256
Cauchy	983.3729	−491.68645	987.3729	
Normal	5642.005	−2821.0025	5646.005	
Laplace	5622.592	−2811.296	5626.592	128
Cauchy	5640.444	−2820.222	5944.444	
Normal	2294.359	−1147.1795	2298.359	
Laplace	2209.32	−1104.66	2213.32	64
Cauchy	1983.201	−991.6005	1987.201	
Normal	594.9543	−297.47715	598.9543	
Laplace	568.3586	−284.1793	572.3586	32
Cauchy	353.777	−176.8885	357.777	

Table 4. *AIC for capital market index.*

DISTRIBUTION	-2 LogL	Log Likelihood	AIC	RESOLUTION
Normal	1037.729	−518.8645	1041.729	
Laplace	840.7146	−420.3573	844.7146	256
Cauchy	491.4878	−245.7439	495.4878	
Normal	4807.809	−2403.9045	4811.809	
Laplace	4838.125	−2419.0625	4842.125	128
Cauchy	4534.695	−2267.3475	4538.695	
Normal	2284.82	−1142.41	2288.82	
Laplace	2101.161	−1050.5805	2105.161	64
Cauchy	1418.954	−709.47735	1422.954	
Normal	139.93	−519.965	1043.93	
Laplace	1198.383	−599.1915	1202.383	32
Cauchy	683.0129	−341.50645	687.0129	

3. Summary, Conclusion and Recommendations

3.1. Summary of Findings

This research work was carried on two stock indexes (namely All s Share Index and capital market index) with data collected from Nigerian stock Exchange (1989-2010).

Wavelet based approach to outlier detection was carried out on the data and Akaike information criterion (AIC) was used to identify the best model for the distribution in this research work. This approach was carried out separately on each of the stock index.

The results obtained from the analysis of all share index (ASI) indicate that there are seventeen aberrant points when the original data was analyzed. The month and years of their occurrence were stated in the interpretation following the analysis, with February 2008 recording the highest index in the Nigerian stock index. The first level of decomposition on the original data picked four aberrant points, with two of them exactly the same as that from the analysis on the original data. The second level of decomposition on the original data picked, two AOs, was found and one of them was traced to the original data which was at 28^{th} position in the original data. For the AIC, it was found that in the second, third and original data, the Cauchy distribution has the minimum AIC, while in the first resolution, the Laplace distribution has the minimum AIC.

The result obtained from the analysis of capital market index indicates there are eleven AOs for the analysis of the original data. The months and corresponding year of occurrence were duly recorded in the interpretation that followed it. The analysis from the first level decomposition of the original data picked five aberrant points, where four of them are exactly the same as that obtained from the analysis of the original data, while one is close to an aberrant observation in the original data. The second level of decomposition, analysis on the original data yield four AOs where two correspond to the original data and the other two closes. While in the third level o f decomposition, analysis on the original data yield three AOs where two are closes to the AOs in the original data and one correspond to the AO in the original data which is at 28^{th} position of the original and it occurs in march 2008. For (AIC), it was found that in all the resolution level, the Cauchy has the minimum AIC.

3.2. Conclusion

It is evident from the analysis of this study that the stock index for all share and capital market are similar, there was time that lower and higher index arises over the period under study. This can be vividly seen from both data analyzed, that there are aberrant location to the left and to the right (that is there are both negative and positive aberrant position) on the analyzed original data. It means there are months that fewer or higher stock index occur in Nigeria economy than expected over the observed years. Again, it can be observed that there was a fall in the months of February, March and July 2008 in both indexes.

The analysis on both the stock indexes showed that the aberrant observations are those where higher and lower index occur. Since the original data at different level of decomposition maintain the same or approximately the same location of AO with that of the analysis for the original data, the importance of wavelet via the multi-resolution has been established. In a situation where there are too many observation points and the data is dyadic, wavelet method of multi-resolution analysis can be applied to detect AOs in such data to reduce the cumbersomeness. The AOs were detected in the months of February, March and July. This is due to some market forces such as inflation and deflations. Which causes the rise and fall of index in the economy, thereby increases the rate of hardship to Nigerians.

However, from the study it is evident that in more recent years, there is a considerable rise in the month of February and which may be due to more people willing to buy stock (demand) than sell it (supply).

From the Akaike information criterion results, it shows that both the data comes from Cauchy distribution because it has the minimum AIC value.

Inflation and deflation causes havoc to the populace and the government has to address that because people's welfare should be government priority. The finding from this study has a significant role to play in the financial field.

3.3. Recommendations

The following are recommended as a possible guide to eradicate (if possible) the menace of inflation and deflation in the economy;

1. Use of contradictory monetary measures: the use of contradictory monetary measures such as increase in bank rate, open market operation, deposit ratio and moral persuasion can help to control inflation.
2. Effective price control system: inflation can also controlled through the use of effective price control system. E.g. price control board by Government officials and the application of rationing to maintain stock index level.
3. Check the activities of hoarders: The activities of hoarders should be checked to prevent increases in index of stocks.
4. Reduction in taxation: This practice enables people to have more money thereby increases their purchasing power and control deflation.
5. Use of open market operation: The central bank does this by purchasing securities from commercial banks to be able to lend money out and increase the volume of money in circulation which lead to stability of stock index.

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